

Equipossibility and Accuracy:

An Old Problem for a New Argument for the Principle of Indifference

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Abstract

For Laplace, equal possibilities entail equal probabilities. However, 'equally possible' better not mean 'equally probable' since this renders the definition circular. Yet, there doesn't seem to be a plausible 'possibility-probability link'. The attempt to justify the principle of indifference by appealing to equipossibility risks either circularity or a lack of justification. Recently, Pettigrew (2016) has provided an argument for the principle of indifference by adapting Joyce's well-known arguments from accuracy (1998/2009). Here, I will argue that Pettigrew's argument implicitly relies on the notion of equipossibility: Just like Laplace, his argument is either circular or unjustified. However, I conclude on a positive note. Pettigrew's argument can be seen as an *explication* of Laplace's argument, and hence of the principle of indifference.

I. Introduction

The principle of indifference tells us that we should assign probability $1/n$ to each of n mutually exclusive and jointly exhaustive events if we do not have further evidence in favor of any of the events. This principle, on the face of it, simply tells us not to treat the events differently if we do not have evidence telling us that they are different. As Norton (2008) notes, principles like the principle of indifference

...are such innocuous principles of evidence as to be near platitudes. They both derive from the notion that beliefs must be grounded in reasons and, in the absence of distinguishing reasons, there should be no difference of belief. How could we ever doubt the notion that, if we have no grounds at all to pick between two outcomes, then we should hold the same belief for each?" (2008, 46)

If the principle is accepted, then we have a clear way to set our priors at the beginning of inquiry and our epistemic pursuits: set it to the uniform distribution over all events.

Agreed: the principle reeks of triviality. Yet, *why* should we think that 'holding the same belief' amounts to assigning the same degree of probability to each event? One way to cash this out arises in Laplace's definition of probabilities:

The theory of chance consists in reducing all the events of the same kind to a certain number of cases *equally possible*, that is to say, to such as we may be *equally undecided about in regard to their existence*, and in determining the number of cases favorable to the event whose probability is sought. The ratio of this number to that of all the cases possible is the measure of this probability, which is thus simply a fraction whose numerator is the number of favourable cases and whose denominator is the number of all the cases possible. (1814, pp. 6-7)

If we have six equally possible cases, then the probability of each of them occurring would simply be one out of six according to this definition – a straightforward application of the principle of indifference, which tells us that we should distribute probabilities equally across the number of cases if we have no further reason to distinguish between the cases other than that we are ‘equally undecided about’. Yet, the question again is this: what do we mean by ‘equally possible’ or ‘equally undecided about in regard to their existence’ such that they dictate equal distributions of probabilities? It better not mean ‘equally *probable*’ since this renders the principle circular, but there doesn’t seem to be a good alternative that can provide a ‘possibility-probability link’, for, as Hájek (2012) notes, possibility does not come in degrees – things are either possible or not, so how do we compare ‘equality’ or ‘degrees’ of possibility? If we do not have an account, then this explanation of the principle of indifference is unjustified. In short, the attempt to justify the principle of indifference by appealing to ‘equally possible cases’ – the notion of ‘equipossibility’ – risks either circularity or a lack of justification.

Recently, however, Pettigrew (2016) has provided an argument for why we ought to adopt the principle of indifference when setting our priors by adapting Joyce’s well-known argument from accuracy (1998/2009). This shall be our starting point.

In this essay, I argue that Pettigrew’s argument risks the same problems that plagued Laplace because it relies on the notion of equipossibility: his argument is either circular or lacks justification. In **II** I rehearse the original Joycean argument for probabilism from accuracy, and in **III** I show how Pettigrew uses the argument from accuracy to the service of the principle of indifference. In **IV** I will show that this argument relies on an implicit assumption of equipossibility; this then leads the argument into the same problems of circularity or a lack of justification. However, in **V** I suggest that, despite its problems, Pettigrew’s account has significant merit, as it can be seen as providing an explication of Laplace’s original claim that equal possibilities demand equal probabilities, by augmenting the claim with arguments from accuracy. Concluding remarks ensue in **VI**.

II. The Original Argument from Accuracy

Joyce (1998/2009) argued that the Kolmogorov axioms of probability are mandated by rationality, because any credence distribution that does not adhere to the axioms of probability is always more inaccurate than some credence distribution that *does* adhere to the axioms of probability; non-probabilistic credence distributions are strictly dominated by some probabilistic credence distribution. Furthermore, there exists no non-probabilistic credence distribution such that this distribution is at least as good in accuracy as a probabilistic distribution in all worlds and better in accuracy than a probabilistic distribution in some world. This in turn suggests that we ought to adopt some probabilistic credence distribution instead of a non-probabilistic one if we think that being accurate – being as close to the truth as possible – is rationally required according to our norms of inquiry.

To put things more rigorously, start by defining a measure of *inaccuracy* at a world $I_w(c(p), o(p))$ which takes as input a credence distribution c and the omniscient distribution o at world w , over the same set of propositions p . c is a possible distribution of probabilities over the set of propositions, and o tell us the actual truth-value of each proposition at that world (hence ‘omniscient’). Intuitively, then, $I_w(c, o)$ measures ‘how far from the truth’ a credence distribution is from the state of affairs – the actual truth-values of each proposition – at that world.

Minimizing I_w then has a natural interpretation of minimizing inaccuracy and thereby maximizing accuracy at that world.

Discussions of such inaccuracy measures often center on the squared distance measure, $\sum_p [c(p) - o(p)]^2$ which is also sometimes called the Brier score.ⁱ Other scores exist, including the logarithmic measure which has some mathematical applications in recent neuroscientific developments such as the Free Energy Principle.

With such an accuracy measure in hand, Joyce then proves a theorem which forms the cornerstone of his argument. Define the set W of all possible worlds w , by considering all possible truth-assignments to the set of all propositions. Then, given a *non-probabilistic* credence distribution c_{nonprob} – one that violates Kolmogorov’s axioms in some way – there exists a *probabilistic* credence distribution c_{prob} – one that does adhere to Kolmogorov’s axioms – which *strictly dominates* that non-probabilistic credence distribution in terms of accuracy: for all w , $I_w(c_{\text{prob}}, o) < I_w(c_{\text{nonprob}}, o)$.

Furthermore, given any probabilistic credence distribution, Joyce (channeled by Pettigrew (2016, 37–38)) also shows that it is not weakly dominated by any other non-probabilistic credence distribution: for *all* probabilistic credence distributions c_{prob} , *there does not exist* any non-probabilistic credence distribution c_{nonprob} such that $I_w(c_{\text{nonprob}}, o) \leq I_w(c_{\text{prob}}, o)$ and for at least one world w $I_w(c_{\text{nonprob}}, o) < I_w(c_{\text{prob}}, o)$.

In words: given any non-probabilistic credence distribution, there exists some probabilistic one that is strictly more accurate (equivalently, less inaccurate) than it. Furthermore, any probabilistic credence distribution is *non-dominated* by any non-probabilistic credence distribution: there are no non-probabilistic credence distributions such that they are at least as accurate as a probabilistic one in all possible worlds and also more accurate than a probabilistic distribution in at least one world. In short: non-probabilistic credence distributions are simply strictly worse off in terms of inaccuracy. Adopting a *dominance* principle of reasoning towards decision-making, Joyce argues that adopting a non-probabilistic distribution given the above is *irrational*.

These two results suggest, insofar as we are *only* concerned with minimizing inaccuracy, that we should adopt a probabilistic credence distribution over a non-probabilistic one (insofar as the two are the only options) *as a matter of epistemic rationality* rather than pragmatic considerations like the Dutch Book. In other words, Joyce’s argument is a ‘nonpragmatic vindication of probabilism’ as advertised.

III. Pettigrew’s Defense of the Principle of Indifference from Accuracy

Pettigrew (2016) adopts much of the formal machinery of Joyce’s argument, and, indeed, can be seen to build off the conclusion that we should adopt probabilism. However, instead of adopting *dominance* reasoning, he argues that we should adopt a *minimax* principle of reasoning: we ought to adopt the credence distribution that *minimizes* the *maximum* inaccuracy possibly had.

The minimax principle is motivated by what he calls ‘cognitive conservatism’, the idea that we should ‘avoid error’: minimizing the worst-case scenario – the maximum inaccuracy – appears to do just that. To be fair, Pettigrew does not really argue for it more than he simply asserts it; he

claims that there is no further grounds for arguing for cognitive conservatism over cognitive radicalism where one aims to maximize the maximum accuracy (and should thereby adopt a *maximax* principle instead of the *minimax* principle): “at this point, it seems to me, we have reached normative bedrock.” (2016, 46) So let us stick with the minimax principle for the sake of argument.

Now, the gist of his argument is that he shows that the unique credence distribution that minimizes maximum inaccuracy is the credence distribution that is prescribed by the principle of indifference.

Pettigrew (2016, 48) defines the uniform probabilistic distribution c_0 – the very same one that the Principle of Indifference prescribes – given a set of finiteⁱⁱ propositions P and a set of possible worlds W generated from the logically consistent truth assignments to P :

$$c_0 = \frac{|\{w \text{ in } W : w(P) = \text{true}\}|}{|\{w \text{ in } W\}|}$$

In words, c_0 assigns to proposition P a probability equivalent to the ratio of its being true to the totality of *possibilia*. The *possibilia* here, as Pettigrew is quick to emphasize, is dependent on the class of propositions we are concerned about; it is not mandated by the Principle of Indifference that all agents adhere to c_0 with respect to some ‘one true’ partition of possibility space or set of propositions.

In the simplest case, consider a single proposition: the coin flip will return heads. There are two possible worlds – two truth-values we can assign to this proposition – and only one in which it is true that the coin flip will return heads. Hence:

$$c_0^{\text{coinflip}} = \frac{1}{2}$$

This is just what the Principle of Indifference prescribes.

Pettigrew (2016, 51–52) aims to prove that this distribution uniquely minimizes maximum inaccuracy: assuming that the measure of inaccuracy I is egalitarian and renders indifference immodest, then for all credence distributions, if $c \neq c_0$, the maximum inaccuracy that c_0 will incur will strictly be lower than the maximum inaccuracy that c will incur:

$$\max_{w \text{ in } W} I_w(c_0, o) < \max_{w \text{ in } W} I_w(c, o)$$

In other words, c_0 uniquely minimizes maximum inaccuracy.

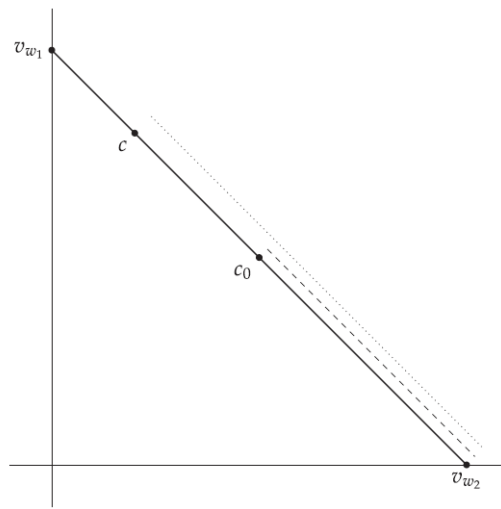
I_w is *egalitarian* if and only if it is only a function of the credence assignments and omniscience function’s assignments to any proposition, and not the content of the proposition itself. This seems fair enough as we are only concerned with the difference between the two distributions’ assignments over those propositions, not the content of those propositions *per se*.

I_w renders indifference *immodest* iff there exists no $c \neq c_0$ such that c is at least as accurate as c_0

in all possible worlds. The argument starts by assuming that there is a possible world where the chances in fact coincide with c_0 . Then, by construction, no other distribution is as accurate as c_0 at that world. Hence, there is no c such that c is as accurate as c_0 at *all* possible worlds.

If the proof is right, then it seems that any $c \neq c_0$ is *irrational* by the lights of the minimax principle since there is another credence distribution one can adopt, c_0 , that goes further in minimizing maximum inaccuracy. Furthermore, *no other* distribution does that.

I won't get into the details of the proof (see Pettigrew 2016) for my argument later will focus on a more foundational aspect of the proof. However, here's a simple example for conceptual clarity, again relying on the simplest case with a single proposition P = "the coin flip will return heads" and two truth-values. The diagram below is borrowed from Pettigrew (2016, 41).



Again, there are two worlds: one in which 'the coin flip will return heads' is true ($o(P)$ at $w_1 = 1$) and one in which 'the coin flip will return heads' is false ($o(P)$ at $w_2 = 0$). Consider the inaccuracy of c_0 where $c_0(P) = 0.5$ at each world when we use the Brier score:

$$\text{At } w_1: \text{Cost} = I_{w_1}(c_0, o) = (0.5 - 1)^2 = 0.25$$

$$\text{At } w_2: \text{Cost} = I_{w_2}(c_0, o) = (0.5 - 0)^2 = 0.25$$

The maximum inaccuracy here is 0.25. Consider the inaccuracy of c at each world, where $c(P = \text{true}) = 0.8$. Then:

$$\text{At } w_1: \text{Cost} = I_{w_1}(c, o) = (0.8 - 1)^2 = 0.04$$

$$\text{At } w_2: \text{Cost} = I_{w_2}(c, o) = (0.8 - 0)^2 = 0.64$$

The maximum inaccuracy here is 0.64 which is higher than the maximum inaccuracy had by c_0 , even though the maximum *accuracy* of c is in fact much higher than c_0 at 0.04. The minimax principle will then tell us that c_0 is rationally preferable to c in this case.

In short, if the above is all right, then it seems we have a rational defense of the Principle

of Indifference from accuracy: the principle of indifference is the unique way to minimize the maximum inaccuracy of our credence distributions, *given* any partition of possibilities.

IV. (Laplace's) Devil in the Details: Equipossibility in Joyce and Pettigrew's Accounts

So far so good. However, as I have noted in the historical case of Laplace in the introduction, there is always a risk of putting things in by hand ('equal possibility') in order to get something out ('equal probability'). Here I will argue that this is precisely what has happened with Pettigrew's argument, but in a much subtler way: there is, again, an assumption of *equipossibility* in the background that is required for his account to take off. This assumption is hidden in the assumption of *veritism*. I show that the same thing occurs in Joyce's argument as well, but that nothing in his argument turns on this issue since, unlike Pettigrew, Joyce did not take his argument to be defending the principle of indifference.

Zooming out, Pettigrew's argument essentially contains this assumption of equipossibility. When considering the maximum inaccuracy of c at each world, there is an additional seemingly trivial requirement that the measure of inaccuracy between a given credence distribution and the omniscient distribution at each world should be comparable to each other world's measure of inaccuracy, such that we can find the maximum inaccuracy *across* worlds.

As we have seen, the strategy for justifying this, for Pettigrew, is (at a very general level) this: Consider a measure such that the only things the measure cares about are the credence ascribed to a proposition, and the truth-values of propositions at a world. Possible worlds are distinguished by the truth-values they ascribe to propositions. Assume *veritism* (Pettigrew 2016, 42): all we care about – the only cognitive value for epistemic decision-making – is truth and how close we are to truth (accuracy). Since there is no particular reason or another to choose one set of truth-value assignments over another from this perspective (they are just truths all the same to anyone with this perspective), there is no particular reason to favor one possible world or another. Each possible world, from the perspective of the measure, is 'just as good' as another. But this, in my view, is essentially assuming *equipossibility*: each possibility is equal to each other.

Here's one way to make this notion of 'just-as-good' concrete: we can understand $I_w(c, o)$ to be a special case of the measure $j_w I_w(c, o)$ which contain an indexical j_w (for 'just-as-good-ness of a world w '), *except* $j_{w1} = j_{w2} = j_{w3} = \dots = j_{wn}$ for n possible worlds; If this condition holds, then each possible world (and the level of inaccuracy at that world) is 'just-as-good as' each other possible world (the exact numerical values don't matter since weights are relative to each other). In this case, the argument goes through for a rational justification of the Principle of Indifference from minimax reasoning. But is there a justification for this special choice of j_w instead of one where $j_{w1} \neq j_{w2} \neq j_{w3} \neq \dots \neq j_{wn}$? After all, there is only one way for each j_w to be equivalent to each other, and *many* ways for them to differ. Note that in cases where j_w differs across worlds, the minimax reasoning doesn't hold. Returning to our toy case above with a single proposition and two possible worlds, suppose that $j_{w1} = 5$ and $j_{w2} = 1$. Then for c_0 :

$$\text{At } w_1: \text{Cost} = 5I_{w1}(c_0, o) = 5(0.5 - 1)^2 = 1.25$$

$$\text{At } w_2: \text{Cost} = I_{w2}(c_0, o) = (0.5 - 0)^2 = 0.25$$

And for c :

$$\text{At } w_1: \text{Cost} = 5I_{w_1}(c, o) = 5(0.8 - 1)^2 = 0.2$$

$$\text{At } w_2: \text{Cost} = I_{w_2}(c, o) = (0.8 - 0)^2 = 0.64$$

Now, the maximum inaccuracy incurred by c_0 is in fact *higher* than c . Minimax prescribes c , *not* c_0 , in this case. The hidden equal weights play a role, however subtle. Setting a uniform j_w amounts to setting equal weights to each world, and this is required for Pettigrew's minimax argument, but is there a good argument for this move?

In statistical mechanics, there is a similar problem: *why should the Lebesgue measure* (a generalization of the uniform measure which applies in continuous settings) be used in measuring the volume of phase space? Why should each partition of the phase space be equal in volume to every other? In that case, however, there is a *physical reason*: it turns out that the (local) Lebesgue measure (or the Liouville measure) is *invariant* along any possible phase flow – any dynamical evolution of a system preserves the Lebesgue measure (a result due to the Liouville theorem); i.e. a volume of phase space, as it evolves along any physically allowed trajectory, maintains its volume.

Is there any argument similar in force that justifies the choice of a uniform j_w in the case of the principle of indifference? Any such argument better not prove too much. For example, one argument that Pettigrew might appeal to might be the thought that his argument for the Principle of Indifference applies only to 'superbabies' (Pettigrew 2016, 35) starting out in inquiry with no evidence whatsoever (except a choice of partition of possibility space, it seems). This superbaby would not know which possible world it is in since it has no evidence: each possible world is 'just as possible' to this superbaby. This seems to justify the equal weights between possible worlds.

But the old problem of this approach being circular-or-unjustified comes back: why should this notion of 'just as possible' be relevant to probabilities at all? On one hand, if 'just as possible' just means 'just as probable', and the baby knows that each possible world is 'just as possible', then we can already apply the Principle of Indifference with no further argument, though our justification for using equipossibility becomes circular. On the other hand, if 'just as possible' is not related to probabilities or credences, then why should this notion be relevant in defending the equipossibility of possible worlds (in the sense of equal weights j_w for each world) when weighing the inaccuracy of our *credence distributions*? Again, a defense of the possibility-probability bridge is required.

What about the appeal to *veritism*, already briefly talked about? We can read it in two ways: to begin, we can read veritism in a stronger and more literal sense as saying that *truth* is all that matters. This is what at first appears to justify an equal weight. If truth is *all* that we care about, as this brand of veritism supposedly claims, then surely each possible world is just as good as the other from the perspective of truth, since each possible world just is a logically consistent assignment of truth-values to a set of propositions. However, consider that we can aim at truth in different ways. We can aim at truth by favoring the *actual world*; this however, would give the actual world an infinitely larger weight than other worlds: why care about the other worlds if we know what is actual? We can also favor the set of worlds which contain a certain number of true propositions or more (especially in finite cases as Pettigrew desires) and weigh other worlds with

fewer true propositions in decreasing order as the number of true propositions decrease. There are many ways to care about truth. Veritism in this stronger sense does *not* require an equal weight for each possible world. Equipossibility is a further assumption.

Pettigrew's veritism, furthermore, is a weaker one, which claims that *accuracy* in the sense laid out by Joyce and Pettigrew is the only thing that matters, as Pettigrew says so himself (2016, 42). What does accuracy mean here? It means to minimize accuracy across possible worlds. How are the possible worlds weighted in terms of accuracy? There is nothing said in this respect and nothing in this respect yet justifies equipossibility. The assumption that there is no reason to care about any world over any other – the assumption of equipossibility – must be put in by hand.

Note that Joyce's argument contains pretty much the same assumption: each world is compared with every other in terms of inaccuracy as well. However, Joyce does not defend the principle of indifference with that assumption, so we cannot fault him for the same problems discussed here. Furthermore, unlike the minimax principle (or minimax-regret as discussed briefly by Pettigrew (2016, 46)) adopted by Pettigrew which includes an *inter-world numerical* component that is affected when the worlds are weighted differently, the dominance principle does *not* have such an inter-world component: dominance is a binary notion while minimax is a matter of degree. For each possible world, a distribution is either dominated or it is not. We then simply consider whether a distribution is (strongly/weakly) dominated or not for each world; no numerical comparisons are required. However, a distribution incurs a maximum inaccuracy *to some degree*, and this is compared between worlds to find the maximum inaccuracy for a distribution *between worlds*, before we compare between distributions. Changing the weights of each world change the considerations and prescription of the principle as shown in the toy case above. In short, the choice of equal weights – the choice of equipossibility – matter for Pettigrew's argument but not for Joyce's argument, so it is no matter that Joyce does not defend the choice of weights.

I have not yet argued *for* different weights between worlds, having only pointed out the possibility of doing so. Instead, all I am doing here is pointing out that Pettigrew's argument for the principle of indifference *crucially* relies on this assumption of equipossibility, and I find no argument defending this position. Of course, what he could do is simply *assume* equipossibility as a fundamental rule – a 'normative bedrock' or something like that – and proceed on with his argument. However, in the absence of any positive argument *for* the choice of equal weights, the entire argument is unmoored. Alternatively, he could take 'equally possible' to mean 'equally *probable*' but using *this* notion of equipossibility to defend the principle of indifference is circular.

Either way of defending the equipossibility assumption seems undesirable: I conclude that Pettigrew's argument for the principle of indifference does not succeed *simpliciter*; it is at best unjustified and at worst circular.

V. Redemption for Laplace?

There is, however, one crucial saving grace for Pettigrew's argument: it improves on and develops Laplace's original argument to much more rigor than Laplace ever did. The original argument by Laplace essentially claims that equal possibilities entail equal probabilities; this seemed intuitive enough. However, there was never much of a clear notion of equal possibility, such that equal

probabilities could come out of the notion in a non-circular fashion.

What Pettigrew has done here, together with my inclusion of the concept of weights for each possible world, is to show how, given an *assumption* of equal weights across possible worlds in the concrete way defined above, we can show fairly rigorously, using the machinery of accuracy-based arguments and the minimax decision-making principle, that we ought to adopt a uniform distribution, i.e. that we ought to adopt the Principle of Indifference and ascribe equal probabilities to each outcome. This *explains* Laplace's original claim in a much more enlightening way and redeems Laplace's claim: *if* we have equal possibility, *then* we have equal probability.

Of course, the notion of equal possibility remains unjustified in both Laplace's and Pettigrew's case, even after being made suitably rigorous via the notion of weights. But perhaps this is the best we can do for the Principle of Indifference.

VI. Conclusion

In a vivid example enacted by Tim Maudlin at a summer school I attended, Maudlin challenged someone – a defender of the principle of indifference – to a bet and told him only that there will be three possible outcomes A, B and C. The defender then bet according to the principle of indifference. Maudlin took out a coin and flipped it.ⁱⁱⁱ Without looking at the outcome of the flip, Maudlin proudly proclaimed: “I win.”

The moral I took from this incident was that not all possibilities are equal, and some justification must be made for the assumption of equipossibility. Pettigrew does not do this, and, as such, his argument for the Principle of Indifference does not succeed.

Endnotes

ⁱ Pettigrew uses $\sum p |c(p) - o(p)|^2$ which isn't the Brier score strictly speaking; the modulus is unnecessary given that everything is raised to an even power. Numerically there are no differences.

ⁱⁱ Pettigrew (2016, 58) notes that adopting c_0 in infinitary cases leads to the usual problems for the principle of indifference like Bertrand's paradox and potential conflicts with additivity.

ⁱⁱⁱ The implication being that the outcomes A, B and C were meant to be heads, tails, and edge.

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